Basic Properties and Facts

Arithmetic Operations

$$ab + ac = a(b+c) \qquad a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc} \qquad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \qquad \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c} \qquad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab+ac}{a} = b+c,\ a \neq 0 \qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

Exponent Properties

$$a^n a^m = a^{n+m} \qquad (ab)^n = a^n b^n$$

$$(a^n)^m = a^{nm} \qquad a^0 = 1,\ a \neq 0$$

$$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}} \qquad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \qquad a^{-n} = \frac{1}{a^n}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$\sqrt[n]{a^n} = a$ if n is odd

$\sqrt[n]{a^n} = |a|$ if n is even

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \geq 0 \qquad |-a| = |a|$$

$$|ab| = |a|\,|b| \qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$|a+b| \leqslant |a| + |b|$ Triangle Inequality

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i = \sqrt{-1} \qquad i^2 = -1 \qquad \sqrt{-a} = i\sqrt{a},\ a \geq 0$$

$$(a+bi) + (c+di) = a + c + (b+d)\,i$$

$$(a+bi) - (c+di) = a - c + (b-d)\,i$$

$$(a+bi)(c+di) = ac - bd + (ad+bc)\,i$$

$$(a+bi)(a-bi) = a^2 + b^2$$

$|a+bi| = \sqrt{a^2+b^2}$ Complex Modulus

$\overline{(a+bi)} = a - bi$ Complex Conjugate

$$\overline{(a+bi)}\,(a+bi) = |a+bi|^2$$

Logarithms and Log Properties

Definition
$y = \log_b(x)$ is equivalent to $x = b^y$

Example
$\log_5(125) = 3$ because $5^3 = 125$

Special Logarithms
$\ln(x) = \log_e(x)$ natural log

$\log(x) = \log_{10}(x)$ common log

where $e = 2.718281828\ldots$

Logarithm Properties
$\log_b(b) = 1$ $\log_b(1) = 0$

$\log_b(b^x) = x$ $b^{\log_b(x)} = x$

$\log_b(x^r) = r\log_b(x)$

$\log_b(xy) = \log_b(x) + \log_b(y)$

$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

The domain of $\log_b(x)$ is $x > 0$

Factoring and Solving

Factoring Formulas
$x^2 - a^2 = (x+a)(x-a)$

$x^2 + 2ax + a^2 = (x+a)^2$

$x^2 - 2ax + a^2 = (x-a)^2$

$x^2 + (a+b)x + ab = (x+a)(x+b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$

$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$

$x^3 + a^3 = (x+a)\left(x^2 - ax + a^2\right)$

$x^3 - a^3 = (x-a)\left(x^2 + ax + a^2\right)$

$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

If n is odd then,

$x^n - a^n = (x-a)\left(x^{n-1} + ax^{n-2} + \cdots + a^{n-1}\right)$

$x^n + a^n =$
$(x+a)\left(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \cdots + a^{n-1}\right)$

Quadratic Formula
Solve $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ – Two real unequal solns.

If $b^2 - 4ac = 0$ – Repeated real solution.

If $b^2 - 4ac < 0$ – Two complex solutions.

Square Root Property
If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities
If b is a positive number

$|p| = b \quad \Rightarrow \quad p = -b$ or $p = b$

$|p| < b \quad \Rightarrow \quad -b < p < b$

$|p| > b \quad \Rightarrow \quad p < -b$ or $p > b$

Completing the Square

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of x, square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs

Constant Function

$y = a$ or $f(x) = a$

Graph is a horizontal line passing through the point $(0, a)$.

Line/Linear Function

$y = mx + b$ or $f(x) = mx + b$

Graph is a line with point $(0, b)$ and slope m.

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$y = a(x-h)^2 + k$ $f(x) = a(x-h)^2 + k$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k).

Parabola/Quadratic Function

$y = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Parabola/Quadratic Function

$x = ay^2 + by + c$ $g(y) = ay^2 + by + c$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$.

Circle

$(x-h)^2 + (y-k)^2 = r^2$

Graph is a circle with radius r and center (h, k).

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k), vertices a units left/right of center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Hyperbola

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k), vertices b units up/down from the center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2 x^2 x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{\cancel{a}+bx}{\cancel{a}} \neq 1+bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax - a$	$-a(x-1) = -ax + a$ Make sure you distribute the "-"!
$(x+a)^2 \neq x^2 + a^2$	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
$\sqrt{x^2+a^2} \neq x + a$	$5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3+4 = 7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$ $(2x+2)^2 = 4x^2+8x+4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parenthesis!
$\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2} = (-x^2+a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$